TRINITY COLLEGE

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS
APPLICATIONS
UNITS 3&4

Section Two:
Calculator-assumed

WA student number: In figures

In words

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (99 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8   **(7 marks)**

A weather balloon is launched from the ground and rises a vertical distance of 450 m during its first minute of flight. In each subsequent minute, it rises 96% as far as in the previous minute.

(a)   Determine the height that the balloon rises in the second minute of flight.   (1 mark)

(b)   Deduce a rule for T_n, the rise in height of the balloon in metres during the nth minute of its flight.   (1 mark)

(c)   Complete the following table to show the rise in height (to the nearest metre) during the specified minutes and display the points from the table on the axes provided.   (4 marks)

Minute of flight, n	5	10	15	20	25
Rise in height, T_n					
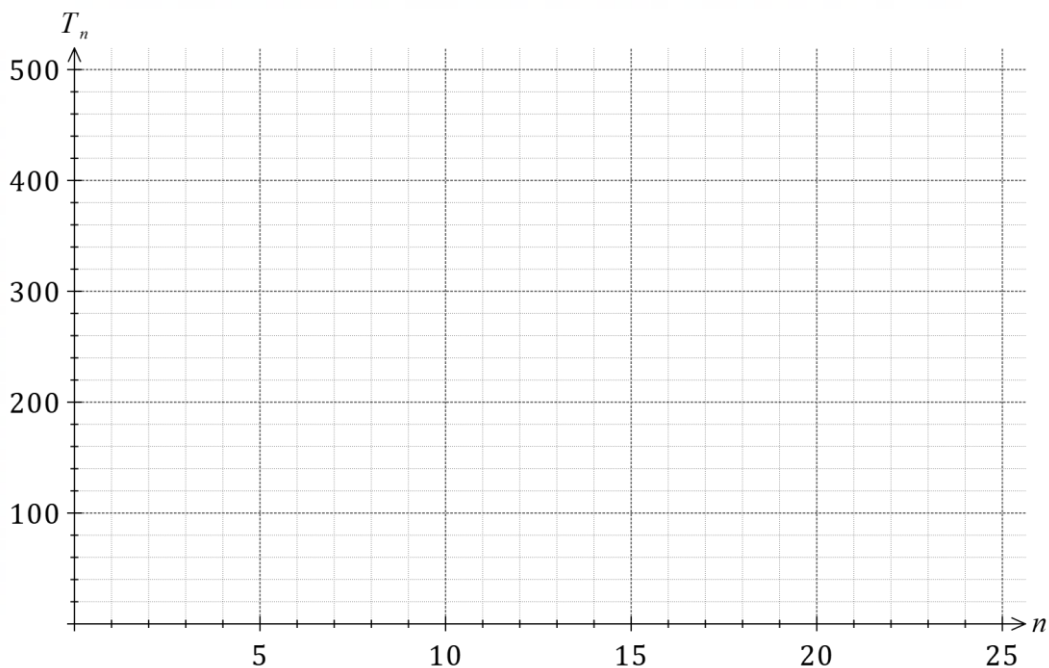
(d)   During which minute is the rise in height of the balloon closest to 60 m?   (1 mark)

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Question 9   **(6 marks)**

The length of cable required to connect routers A to F is shown in the table below. It is not possible to connect all routers to each other and such pairs are shown with a dash. The network cable used costs $9 per metre.

	F	E	D	C	B
A	8	6	–	7	4
B	–	–	7	8	
C	–	5	4		
D	8	3			
E	7				

(a) Use the vertices below to complete, in the plane, an undirected weighted graph that represents the information in the table.   (2 marks)
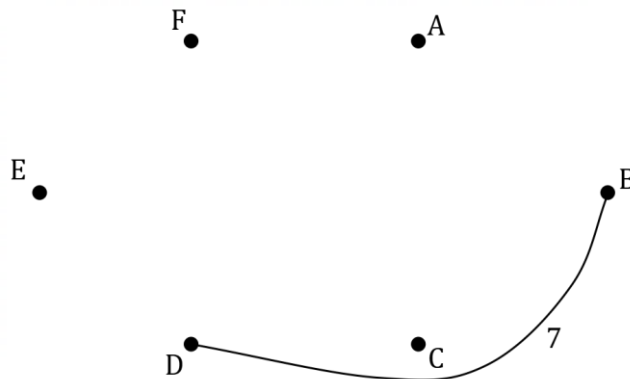
(b) Use the vertices below to draw the minimum spanning tree for the graph in part (a).   (2 marks)
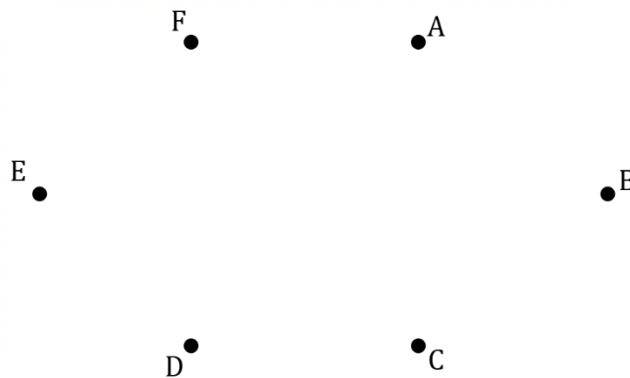
(c) If it is no longer possible to connect routers A and B, determine the cost of cable to connect the routers using the new minimum spanning tree.   (2 marks)

Question 10   **(7 marks)**

The number of rainy days at a weather station were recorded and a summary by quarter for a 3-year period is shown in the table below.

Year	2020				2021				2022			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
Days	18	10	23	34	15	9	24	38	23	A	22	33

(a)   Determine

     (i)   the 3-point moving average for the fourth quarter of 2020.   (1 mark)

     (ii)   the 4-point centred moving average for the second quarter of 2021.   (1 mark)

     (iii)   the value of A in the table, given that the 5-point moving average for the first quarter of 2022 is 23.   (2 marks)

(b)   Briefly explain the purpose of calculating moving averages for time series data.   (1 mark)

(c)   Which is the most appropriate type of moving average for use with the data in the table above? Justify your choice.   (2 marks)

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Question 11   **(7 marks)**

A finance company offers three loan products to borrowers, with interest compounded at different intervals:

Product	X	Y	Z
Annual interest rate	8.68%	8.70%	8.77%
Compounding period	Daily	Monthly	Quarterly

(a)   Which product would minimise the interest payable on a loan taken out for two years?
       Justify your answer.   (3 marks)

(b)   Determine the interest that a borrower will have to pay on a loan of $2500 for nine months
       using product Z.   (2 marks)

(c)   A borrower took out a short-term loan of $2200 dollars using product X. If they paid $19.44
   interest on the loan, determine the duration of the loan in days.   (2 marks)

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Question 12   **(10 marks)**

The capacity of a solar farm is the maximum amount of power the farm is the designed to produce. The table below shows the power capacity in megawatts of ten Australian solar farms (labelled A to J) and the total power each generated in gigawatt-hours over the course of October last year.

Solar farm	A	B	C	D	E	F	G	H	I	J
Capacity, c MW	31	80	106	48	121	63	140	126	54	97
Generation, g GWh	3.5	14	17	8.5	18	7.5	22	23	6.1	13
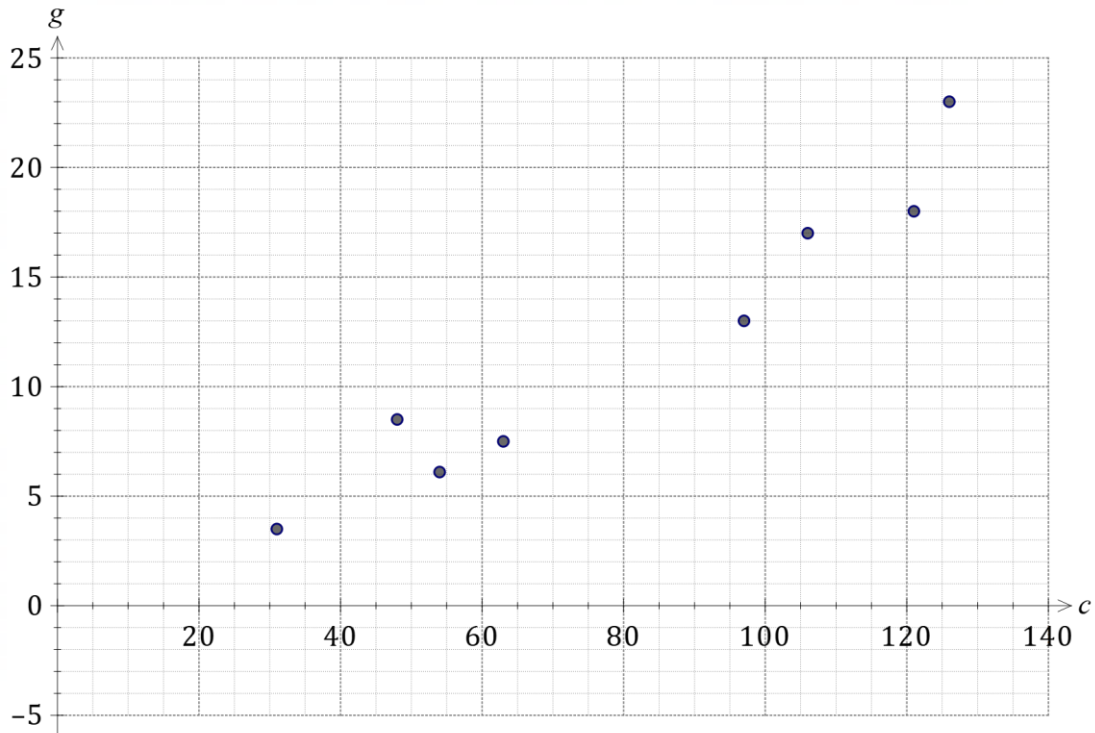
(a)   Use data from the table to plot the points B and G on the graph.   (1 mark)

(b)   Determine the equation of the least-squares line in the form $g = ac + b$, where a and b are constants, and state the value of the correlation coefficient.   (2 marks)

(c)   Draw the least-squares line on the graph.   (2 marks)

See next page

(d)   Interpret the gradient of the least-squares line in the context of the question.   (1 mark)

(e)   What percentage of the variation in power generation can be explained by the variation in power capacity?   (1 mark)

Australian solar farm K, with a capacity of 74 MW, was also operating last October.

(f)   Predict the total power generated by solar farm K that month.   (1 mark)

(g)   Give two reasons why the prediction in part (f) is reliable.   (2 marks)

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Question 13 (12 marks)

A small business has organised a loan of $6250 to purchase a new machine. To pay off the loan, the business will make repayments of $233.08 at the end of each month. The spreadsheet below shows how the balance of the loan will reduce in the first few months.

Month (n)	Opening balance (T_n)	Interest	Repayment	Closing balance
1	6250.00	46.25	233.08	6063.17
2	6063.17	44.87	233.08	5874.96
3				

(a) Show that the loan interest rate is 8.88% per annum. (2 marks)

(b) Complete the third row of the spreadsheet. (3 marks)

The recursive rule for the opening balance is $T_{n+1} = aT_n - b$, $T_1 = c$, where T_n is the opening balance of the loan in month n.

(c) Determine the values of the constants a, b and c. (2 marks)

(d) Determine how many months it will take the business to fully repay the loan. (1 mark)

See next page

(e)   Calculate the total interest paid by the business once the loan has been fully repaid.

(2 marks)

(f)   If the interest rate was doubled but the loan repayments remained the same, would the time to repay the loan also double? Justify your answer.   (2 marks)

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**Question 14**　　　　　　　　　　　　　　　　　　　　　　　　**(11 marks)**

National chess organisations rate all players who are registered to compete in tournaments. When a higher rated player competes against a lower rated player, the higher rated player is expected to win. The table below shows the average rating of registered South African players of different ages.

Age, x years	10	11	12	13	14	15	16	17	18
Average rating, y	1216	1196	1218	1292	1253	1282	1319	1306	1354

The data has a correlation coefficient of 0.92 and the equation of the least-squares line is $y = 17.9x + 1020$.

(a)　Explain the significance of the correlation coefficient in the context of the question.

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(2 marks)

(b)　Predict the average rating of 19-year old registered South African players and briefly discuss the validity of your prediction.　　　　　　　　(3 marks)

(c)　It is unlikely that simply getting older causes players ratings to increase. Suggest another variable that is more likely to cause the rating increase.　　　(1 mark)

(d)   Complete the residual plot for the data on the axes below.   (3 marks)
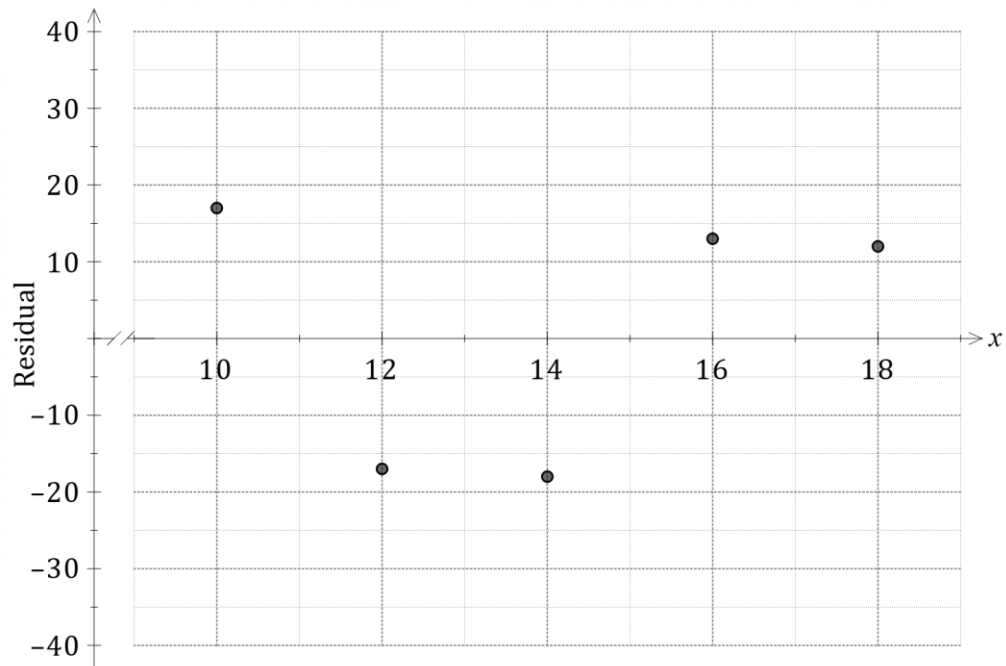
(e)   Use the residual plot to comment on the appropriateness of fitting a linear model to the
       data.   (2 marks)

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Question 15　　　　　　　　　　　　　　　　　　　**(9 marks)**

The graph represents a network of tracks in a park between nine locations labelled A, B, C, D, E, F, G, H and K. The number on each edge is the length, in hundreds of metres, of the corresponding track. The total length of all tracks in the network is 30 100 m.
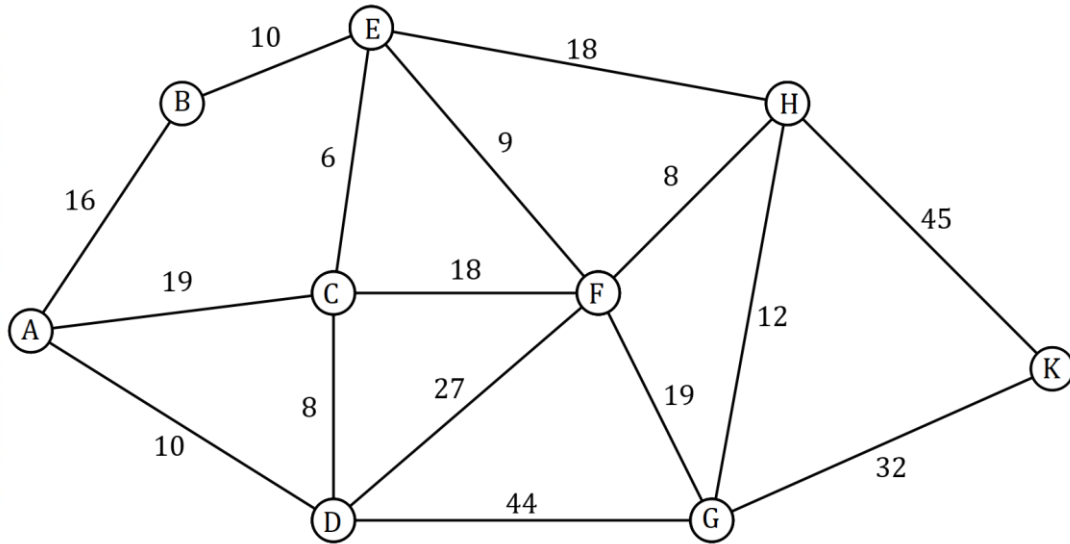
(a)　Determine the shortest distance along these tracks a person at location D must travel to reach location B and state the corresponding route.　　　　　　**(2 marks)**

(b)　Determine the shortest path from location A to location K and state the length of this path.　　　　　　**(4 marks)**

A copy of the network from the previous page is shown below.
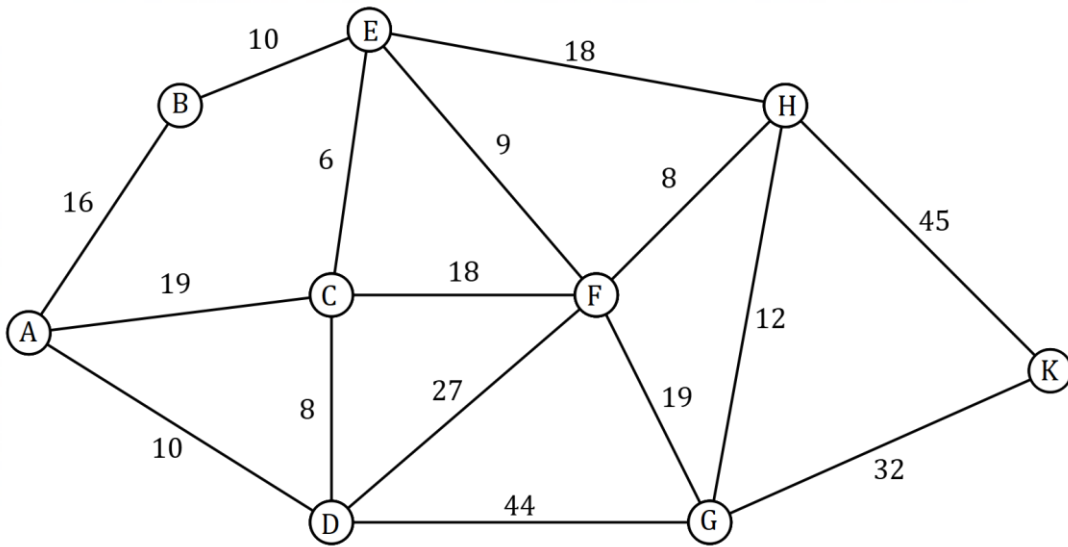
(c)   Every track in the park is to be inspected by a ranger who must start and end the inspection at location A and travel along each track at least once. Determine the least distance the ranger must travel to perform this task. Justify your answer.   (3 marks)

Question 16   **(7 marks)**

The table below shows the nine activities that must be completed in order to assemble a piece of flat-packed furniture, together with the time required and immediate predecessors for each activity involved in the project.

Activity	A	B	C	D	E	F	G	H	K
Time (minutes)	7	7	6	9	6	5	4	5	4
Immediate predecessors	–	–	B	B	A, C	A, C	E	E, F, D	D

(a)   Complete the project network below, showing all activities and their times.   (3 marks)
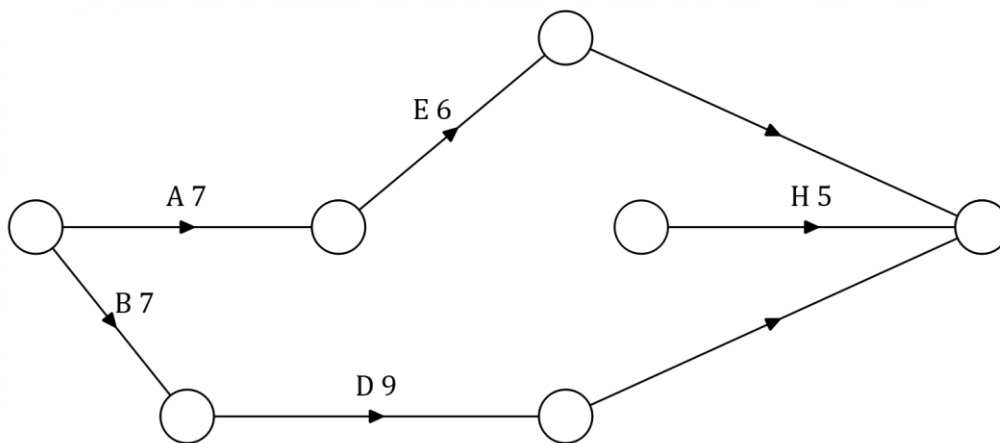
(b)   Determine the critical path and the minimum completion time for the project.   (2 marks)

(c)   A redesign of the flat-packed furniture will reduce the time required for Activity C to just two minutes. What effect will this have on the critical path and the minimum completion time?   (2 marks)

Question 17 **(7 marks)**

The number of residential solar panel installations carried out by a company during each of the first nine months of the year 2020 is shown in the table below.

Quarter	Month	t	Installations n	Quarterly average	Installations as percentage of average
1	January	1	132		85.2
1	February	2	171	155	111.3
1	March	3	162		104.5
2	April	4	148		87.1
2	May	5	188	A	110.6
2	June	6	174		102.4
3	July	7	156		86.7
3	August	8	198	180	B
3	September	9	186		103.3

The long-term trend y in the number of installations can be modelled using $y = 3.77t + 149.5$, and the seasonal index for the last month of every quarter is 1.034.

(a) Calculate the value of A and the value of B in the table. (2 marks)

(b) Clearly show why the seasonal index for the first month of every quarter is 0.863. (1 mark)

(c) Determine the deseasonalised number of installations carried out in March 2020.

(2 marks)

(d) Assuming that existing trends and seasonality continue, predict the number of installations carried out by the company during April 2021. (2 marks)

See next page

Question 18   **(7 marks)**

Lucky has a savings account with a balance of $402 and plans to deposit $11 into it on the first day of every month. Interest on the balance of the account, at a rate of 0.6% per month, is added on the last day of every month.

To model the balance of this account, Lucky uses the recurrence relation $T_{n+1} = 1.006T_n + 11$, $T_0 = 402$, where T_n is the account balance in dollars just after he has made his n^{th} deposit.

After the ninth deposit, the balance of the account is $525.65.

(a)   Determine the account balance after the 10th deposit.   (1 mark)

(b)   Does the sequence generated by the recurrence relation have a long-term increasing, decreasing or steady-state solution? Justify your answer.   (2 marks)

(c)   Consider the case where the account paid no interest. Determine the regular monthly deposit Lucky should make into his savings account so that he still achieves the same balance as in part (a) after making 10 deposits.   (2 marks)

(d)   Consider the case where Lucky made no monthly deposits. Determine the monthly interest rate required for his savings account so that he still achieves the same balance as in part (a) after 10 compounding periods.   (2 marks)

Question 19   **(9 marks)**

Penny has decided to take some time off work and has $72 500 in an investment account that attracts interest at a rate of 4.65% compounded monthly. She will withdraw living expenses of $3900 from the account at the end of each month, just after interest is added.

(a)   Determine the balance of her account after one year, when Penny has made 12
        withdrawals.   (3 marks)

(b)   Determine the total interest Penny's account will earn during the first year.   (2 marks)

(c)   For how many months will Penny be able to withdraw $3900?   (1 mark)

(d)   Penny contemplated receiving a quarterly perpetuity from her investment account to fund
        her time off work. Determine the quarterly amount that she would receive.   (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

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